Finite width effects in the model of unstable particles with random mass

V.I.Kuksa

Institute of Physics, Rostov State University,

pr. Stachki 194, Rostov-on-Don, 344090 Russia,
E-mail address: kuksa@list.ru, ph.8632-202874

Abstract

A phenomenological model of unstable particles based on uncertainty principle is discussed in quantum field approach. We show that the simplest quantum field description of mass uncertainty makes it possible to account finite width effects for particles with large decay widths.

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1. Introduction

To test the Standard Model predictions in processes with participation of W - , Z - bosons and t - quark one should take into account finite width or instability effects [1]. At last time these effects have been evaluated in the cases of $t \to WZb$ [2] and some hadron decays [3] - [5]. Such calculations strongly depend on the unstable particle conception, which is the subject of intensive discussion now.

Quantum field description of unstable particles (UP) has been discussed during last five decades [6] - [13]. Two directions of investigation were founded in these works – "propagator" and "spectral function" approaches. The conventional way to take into account the instability effects consist in the Dyson resummation of self-energy in "propagator" approach [10] - [13]. This procedure is the most direct and consistent quantum field description of UP [12] - [14] and has got development in numerous works [15] - [30]. However the realization of this program runs into problems connected with the requirements of being unitary, gauge invariance and with procedure of renormalization in perturbation theory [13, 18, 20, 22, 39, 29]. Precise and consistent definitions of mass and width for UP have been notoriously difficult yet, caused by above mentioned problems. The principal source of methodical difficulties is connected with the fact that UP's have finite lifetime and therefore lie somewhat outside the traditional formulation of quantum field theory [13, 28]. As a result some phenomenological methods are used in description of UP and resonance lines.

The second direction was formulated by Matthews and Salam in [8, 9] and is connected with Lehmann (spectral) representation [6]. It is based on the uncertainty principle for unstable quantum system, which leads to the uncertainty relation for mass of unstable particle in the rest frame [8, 10]:

$$\delta m \; \delta \tau \sim 1 \quad \longrightarrow \quad \delta m \sim \Gamma_{tot} \; (c = \hbar = 1). \tag{1}$$

For UP with large Γ_{tot} the large value of mass uncertainty leads to noticeable modification of decay properties or to, so called, "mass smearing" effects. The value of these effects were calculated in phenomenological way for B^0 and Λ^0 decays [3], [31] - [33] and for decay channel $t \to ZWb$ in convolution and decay-chain method [2].

In this work the model realization of "mass smearing" idea (mass uncertainty) is represented in the quantum field framework. The main element of the proposed model is the simplest generalization of field operator function which describes the UP as particles with non-fixed masses. It was shown that the model is convenient and simple tool for evaluation of finite width effects. The model predictions are in agreement with experiment and

describes some peculiarities in generation and decay processes of particles with large total widths. Short version of the model was represented in [5] where, in particular, large instability effects for some hadron decays were discussed in details.

2. The model of unstable particles with random mass parameter

In accordance with uncertainty principle the model field operator function is represented as superposition of ordinary ones weighted by some model function $\omega(\mu)$. For simplicity we consider scalar field:

$$\varphi(x) = \int \omega(\mu)\varphi_\mu(x)\, d\mu, \tag{2}$$

where $\varphi_\mu(x)$ is usual field operator, which describes the state with fixed mass $m^2 = \mu$:

$$\varphi_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \varphi_\mu(k)\delta(k^2 - \mu)e^{ikx}\, d^4k. \tag{3}$$

In (2) and (3) parameter μ has the status of m^2 because $\mu = k^2$. For stable particle $\omega(\mu) = \delta(\mu - M^2)$ and for UP $\omega(\mu)$ describes the finite width or "mass smearing" effects. The expressions (2) and (3) describes ensemble of unstable particles and can be interpreted as a decision of motion equation which follows from model Lagrangian for "free" fields:

$$L(\varphi(x)) = \frac{1}{2} \int |\omega(\mu)|^2 (\partial_k \varphi_\mu(x)\partial^k \varphi_\mu(x) - \mu\, \varphi_\mu(x)\varphi_\mu(x))\, d\mu. \tag{4}$$

Thus, we discuss the approach where "spreading" caused by interaction of UP with decay channels is described by wave packet (2) but corresponding field operator function has the status of initial "free" field. Such approach, as it will be shown in this section, is some phenomenological alternative to propagator renormalization method.

Commutation relations have an additional δ - function of parameter μ:

$$[\varphi_\mu^-(\vec{k}), \dot{\varphi}_{\mu'}^+(\vec{k'})]_- = \delta(\vec{k} - \vec{k'})\delta(\mu - \mu'), \tag{5}$$

where creation and annihilation operators are defined as $\varphi_\mu^{+,-}(\vec{k}) = \varphi_\mu^{+,-}(k)/\sqrt{2k^0}$ and $k^0 = \sqrt{\vec{k}^2 + \mu}$. Relation (5) means additional assumption: the acts of creations and annihilations of particles with various μ don't interfere. So, the parameter μ has status of physically distinguishable value as m^2. Now we'll show that (2) , (3) and (5) lead to Lehmann type

spectral representation of causal Green function. In coordinate representation from (5) and (3) it follows:

$$[\varphi_\mu^-(x), \dot{\varphi}_{\mu'}^+(y)]_- = \delta(\mu - \mu') \frac{1}{i} D_\mu^-(x - y). \tag{6}$$

In (6) $D_\mu^-(x - y)$ is Pauli – Jordan function defined as:

$$D_\mu^-(x - y) = \frac{i}{(2\pi)^3} \int \frac{d\vec{k}}{2k_\mu^0} e^{-ik(x-y)}, \tag{7}$$

where $k_\mu^0 = \sqrt{\vec{k}^2 + \mu}$. The function $D_\mu^+(x - y)$ is defined in analogy with (7). Taking into account (2) we can get spectral representation of Pauli – Jordan function:

$$[\varphi^-(x), \dot{\varphi}^+(y)]_- = \frac{1}{i} D^-(x - y) = \frac{1}{i} \int \rho(\mu) D_\mu^-(x - y) \, d\mu, \tag{8}$$

where $\rho(\mu) = |\omega(\mu)|^2$ is model probability density.

The causal Green function

$$i\langle T[\varphi(x)\varphi(y)]\rangle_0 = D^C(x - y), \tag{9}$$

can be expressed through the Pauli – Jordan functions:

$$D^C(x - y) = \theta(x^0 - y^0)D^-(x - y) - \theta(y^0 - x^0)D^+(x - y). \tag{10}$$

Using (8) we get spectral representation of casual function

$$D^C(x) = \int \rho(\mu) D_\mu^C(x) \, d\mu, \tag{11}$$

where:

$$D_\mu^C(x) = \frac{1}{(2\pi)^4} \int \frac{e^{-ikx}}{\mu - k^2 - i\varepsilon}. \tag{12}$$

In momentum representation from (11) and (12) it follows

$$D^C(k) = \int \frac{\rho(\mu) \, d\mu}{\mu - k^2 - i\varepsilon}. \tag{13}$$

There is one undetermined yet element in the model – the probability density $\rho(\mu) = |\omega(\mu)|^2$. To find $\rho(\mu)$ we have identified the model Green function $D^C(k)$ with standard renormalized propagator by means of analytical continuation to complex plane $k^2 \to z$:

$$\frac{1}{z - M_0^2 - \Sigma(z)} \longleftrightarrow \int_{\mu_0}^\infty d\mu \frac{\rho(\mu)}{z - \mu} = D^C(z), \tag{14}$$

where $\Sigma(k^2 \pm i\epsilon) = Re\Sigma(k^2) \mp iIm\Sigma(k^2)$ [12] and μ_0 is threshold. From (14) and Cauchy theorem we have:

$$D^C(\mu + i\epsilon) - D^C(\mu - i\epsilon) = \oint_\Gamma dz \frac{\rho(z)}{\mu - z} = -2\pi i\rho(\mu), \tag{15}$$

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where Γ is the contour with cut along real axis from μ_0 to positive infinity. On the other hand, right side of the expression (15) can be represented with help of identification (14) in the form:

$$D^C(\mu + i\epsilon) - D^C(\mu - i\epsilon) = \frac{-2iIm\Sigma(\mu)}{[\mu - M_0^2 - Re\Sigma(\mu)]^2 + [Im\Sigma(\mu)]^2} \,. \tag{16}$$

From (15) and (16) one can easily get the expression for probability density:

$$\rho(\mu) = \frac{1}{\pi} \frac{Im\Sigma(\mu)}{[\mu - M_0^2 - Re\Sigma(\mu)]^2 + [Im\Sigma(\mu)]^2} \,. \tag{17}$$

From (17) it follows that model weight function $\omega(\mu)$ can be defined in the form:

$$\omega(\mu) = \frac{1}{\sqrt{\pi}} \frac{\sqrt{Im\Sigma(\mu)}}{\mu - M_0^2 - \Sigma(\mu)} \tag{18}$$

At peak range $\mu \approx M_0^2 + Re\Sigma(M^2)$ we have usual Breight–Wigner type (Lorentzian) distribution:

$$\rho(\mu) \approx \frac{1}{\pi} \frac{M\Gamma}{(\mu - M^2)^2 + M^2\Gamma^2} \,, \tag{19}$$

where $M^2 = M_0^2 + Re\Sigma(M^2)$ and $M\Gamma = Im\Sigma(M^2)$. The identification (14) establishes connection between discussed model and "propagator" method.

3.   Generation and decay of unstable particles

The state vector is determined in a standard way with additional variable μ:

$$|\vec{k}, \mu\rangle = \dot{\varphi}_\mu^+(\vec{k})|0\rangle; \quad \varphi_\mu^-(\vec{k})|0\rangle = 0. \tag{20}$$

Then the transition amplitude contains $\omega(\mu)$ as a result of commuting

$$[\varphi^-(x), \dot{\varphi}_\mu^+(\vec{k})]_- = \frac{1}{(2\pi)^{3/2}} \frac{\omega(\mu)}{\sqrt{2k_\mu^0}} \exp[-ikx], \tag{21}$$

where $k_\mu^0 = \sqrt{\vec{k}^2 + \mu}$ and $\dot{\varphi}_\mu^+(\vec{k})$ – creation generator of state vector. From this result it follows that transition amplitude is a product of weight function $\omega(\mu)$ and expression for amplitude A^{st}, calculated in standard (conventional) way with fixed mass $\mu = m^2$:

$$A(k, \mu) = \omega(\mu) A^{st}(k, \mu). \tag{22}$$

In (22) the value k stands for all kinematics variables. The expression for differential probability of transition is

$$dP(k, \mu) = |\omega(\mu)|^2 |A^{st}(k, \mu)|^2 \, d\mu, \tag{23}$$

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where $\rho(\mu) = |\omega(\mu)|^2$ is the model probability density. From (23) we can straightway get the formula for decay width in form of standard one weighted by $\rho(\mu)$:

$$\Gamma(\bar{\mu}, \sigma) = \int_{\mu_d}^{\mu_u} \rho(\mu; \bar{\mu}, \sigma) \, \Gamma^{st}(\mu) \, d\mu \,, \tag{24}$$

where μ_d and μ_u depend on threshold and total energy of processes. In (24) for simplicity we represent $\rho(\mu)$ by some two-parametric distribution function $\rho(\mu; \bar{\mu}, \sigma)$ with mean value $\bar{\mu} \approx M^2$ and mean square deviation $\sigma \approx \Gamma/2$. In general case we should to determine $\rho(\mu)$ with help of (18) or (19) but for rough evaluation we can approximate $\rho(\mu)$ by phenomenological two-parametric distribution function. In this case our model reduces to the phenomenological account of finite width effect. The expression (24) can be applied when UP with large total width is in both initial or final state. Moreover, it can be easy generalized to the case when there are two or more such particles. When $\rho(\mu) = \delta(\mu - M^2)$ we get usual result in fixed mass approach. In general case (24) leads to modification of phase space, threshold "smearing" and s – dependence of width.

To illustrate the deviation of model predictions from conventional ones we choose Gaussian approximation for $\rho(\mu)$ due to suitable asymptotic behavior in infinity (large μ_{max}). In a case of heavy boson decay to two fermions when $M_f \ll M_V$ and $\bar{\mu} = M_V^2$, $\sigma = \Gamma_V^{tot}/2$ we get

$$\Gamma^M(\bar{\mu}, \sigma)/\Gamma^{st}(\bar{\mu}) \approx 1 + 3\Gamma_{tot}^2/4M^2 \,, \tag{25}$$

where Γ^M and Γ^{st} are model and standard predictions for discussed partial widths. Decay low is subjected to analogous modification. For small time $t/\tau(\bar{\mu}) \ll 1$ we have deviation from exponential low:

$$\frac{N(t)}{N_0} \approx 1 - (1 + \frac{3\Gamma_{tot}^2}{4M^2})\frac{t}{\tau(M)}. \tag{26}$$

It should be noted that the model modification of decay low differs from one discussed in [34] - [37]. From (25), (26) it follows that the model corrections in the discussed case are rather small ($\sim \Gamma_{tot}^2/M^2$). However in the cases of near threshold decays ($m_1 + m_2 \approx M$) this corrections are very large (see the next section).

4. Experimental test of the model

Finite width effects in decays of fundamental UP with large Γ_{tot}, such as Z, W - bosons and t - quark, should be taken into account in precise measurements. The deviation of $Br^M(Z \to f\bar{f})/Br^{st}(Z \to f\bar{f})$ from unity according to (25) is equal to $3\Gamma_{tot}^2/4M_Z^2 \approx 6*10^{-4}$. So, the effect of instability in such channels gives corrections an order of 0.1% that is an

order of least experimental errors in Z - physics and much less than errors in W - physics. We need more precise both experimental data and theoretical calculations in Standard Model.

Significantly more large effects of instability (finite width) take place in near threshold processes when $(M_i - M_f) \sim \Gamma_{tot}$, for example in decays $Z \to Wbc$ or $t \to WZb$. Last process was discussed in detail in [2] without and with account of instability effects. These effects were evaluated in the frame of so called "decay-chain" method and "convolution" method where double weighting and two distribution of invariant mass (for W and Z) was applied in formal analogy with our mass parameter distribution $\rho(\mu)$. It was fond in [2] that

$$10^{-6} < Br(t \to WZb) < 10^{-5} \text{ for } (170 < m_t < 180)\,Gev,$$

while usual calculation with fixed masses gives us $Br(t \to WZb) = 0$ for $M_t < M_W + M_Z + M_b$ and $Br(t \to WZb) = 10^{-7}$ for $M_t = 180\,Gev$. Unfortunately, even result obtained with account of widths effects makes the observation of this decay channel at LHC very difficult.

Instability effect more accessible for observation can occur in decay channel $Z \to Wbc$ and should be taken into consideration in precision Z - physics. The value of the model correction to $Br(Z \to Wbc)$ mainly caused by modification of phase space. For discussed process the phase space $R(M_i)$ in nonrelativistic case can be expressed in the form [38]:

$$R(M_i) \approx \frac{\pi^3}{2} \frac{\sqrt{M_W M_b M_c}}{(M_W + M_b + M_c)^{3/2}} (M_Z - M_W - M_b - M_c)^2. \qquad (27)$$

Using double Gaussian (for simplicity) weighting with parameters $\bar{\mu}_Z = M_Z^2$, $\sigma_Z = \Gamma_Z^{tot}/2$ and $\bar{\mu}_W = M_W^2$, $\sigma_W = \Gamma_W^{tot}/2$ from (27) we get:

$$\frac{Br^M(Z \to Wbc)}{Br^{st}(Z \to Wbc)} \approx 1 + \frac{\sigma_Z^2 + \sigma_W^2}{(M_Z - M_W - M_b - M_c)^2} \approx 1.1 \qquad (28)$$

So, the rough model evaluation of correction caused by finite width effect in rare decay channel $Z \to Wbc$ gives the value $\approx 10\%$. This correction should be taken into account when precision measurements are compared with theoretical prediction.

The effects of "mass smearing" have large value in the processes of generations and decays of hadrons with large total widths. Hadrons are not fundamental particles and quantum field approach can't be applied in general case. But "mass smearing" effect follows from fundamental uncertainty principle and takes place at various hierarchy levels. Proposed model does not describe hadron decays but gives us a simple way to evaluate instability effects as correction to traditional calculations.

The first punctual evaluations of finite width effects in heavy hadron decays were fulfilled in [3], [31] - [33]. The phenomenological Breight-Wigner type weighting of expressions for widths was applied in these works for decays $B^0 \to D^- \rho^+, D^- a_1^+$; $B_s^0 \to D^- a_1^+$ and

$\Lambda_b^0 \rightarrow \Lambda_c^+ a_1^-$. The results of calculations reveal that contributions of "mass smearing" effects are large – from 20% to 40%, and its accounting improves considerably the conformity of experimental data and theoretical predictions.

One of the most pure effect of "mass smearing" in hadron physics takes place in decay channels $\phi(1020) \rightarrow K^+K^-, K_L K_S$. The ratio of branchings does not depends on hadron factors in good approximation and is equal to the ratio of phase space [4]:

$$k = \frac{Br(\phi \rightarrow K^+K^-)}{Br(\phi \rightarrow K_L K_S)} = \frac{g_+^2}{g_0^2} (\frac{1 - 4m_+^2/m_\phi^2}{1 - 4m_0^2/m_\phi^2})^{3/2} . \tag{29}$$

There is a discrepancy between experimental and theoretical values of k when $g_+^2 = g_0^2$, which was discussed in [4]:

$$k^{exp} = 1.456 \pm 0.033, \quad k^{th} = 1.528$$

Various corrections to k^{th} have been evaluated in [39] but discrepancy has increased only (Fermi "gold rule" puzzle). The model prediction k^M depends on μ_{max} and in Breight-Wigner type approximation for $\rho(\mu)$:

$$k^M = 1.42 - 1.49 \text{ when } \mu_{max} = (1 - 3) \, Gev, \quad k^M = k^{exp} \text{ when } \mu_{max} = 2 \, Gev.$$

So, the model can resolve discussed problem with help of reasonable assumption concern μ_{max}. Analogous result was received in [4] with assumption of phase space s-dependence, which is similar to convolution method [2].

The decay channel $f_o(980) \rightarrow K\bar{K}$ is the example of "mass forbidden" one $(M_{f_o} < 2M_K)$ and we have the effect of "threshold smearing". Model prediction for the ratio of forbidden and dominant branchings $(g_K \sim g_\pi)$:

$$Br(f_0 \rightarrow K\bar{K})/Br(f_0 \rightarrow 2\pi) \sim 0.1$$

This rough estimation should be considered as the conformity of theoretical prediction and experimental indication of channel $Br(f_0 \rightarrow K\bar{K})$ ("seen", [40]).

There are many examples of hadron decays with large total width and of near threshold decay channels. The determination of hadron's probability density $\rho(\mu)$ in quantum field approach is limited due to its composite structure. However, as it had been shown in [5], calculation of instability effects can be done for some hadron decays with high accuracy (2 - 3 %). This problem needs more detailed consideration with help of phenomenological methods.

5.   Conclusion

Proposed model of UP is the simplest phenomenological realization of uncertainty principle in the framework of quantum field theory. The model calculation of decay rates is in formal analogy with the "convolution" type treatment but model structure contains phenomenological elements at more fundamental level. Quantum field approach restricts application of the model to hadron decays but successfully describes some peculiarities connected with finite width effects. The model does not contradict to experimental data on decays of fundamental particles and is in quantitative agreement with the data on some hadron decay channels.

The principal element of discussed model is the wave packet (2) which describes initial or final "free" state vector with commutation relations (5). This packet is the result of model accounting of interaction connected with decay channels, which leads to "spreading" of mass. All information about this interaction enter to the probability density $\rho(\mu)$. The status of random mass parameter μ is determined by dispersion condition $\mu = k^2$ and relations (5) as physical random mass squared of UP. This interpretation arises the question on physical meaning of unstable particle in real and virtual states.

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